Software and Systems Engineering

CPE3202

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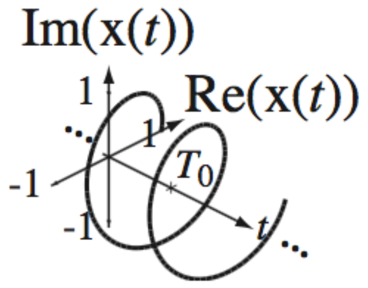
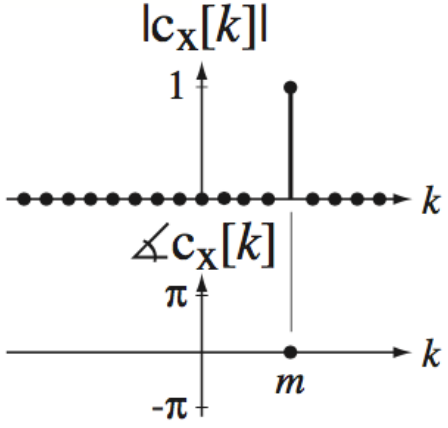
Fourier Transform

Continuous-Time Fourier Series Pairs (CTFS)

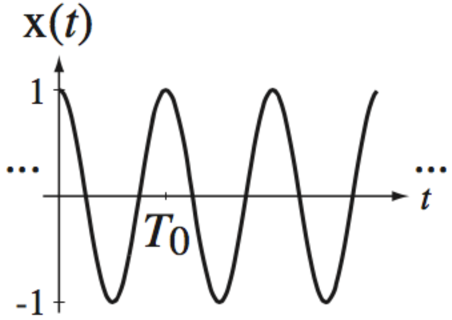
$$\mathrm{x}(t) = \sum_{k=-\infty}^{\infty} \mathrm{c_x}[k] e^{j2\pi kt/T} \xleftrightarrow[T]{\mathcal{FS}} \mathrm{c_x}[k] = \frac{1}{T}\int_T \mathrm{x}(t) e^{-j2\pi kt/T}\, dt$$



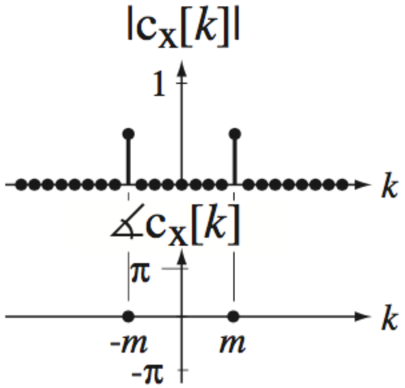
$$e^{j2\pi t/T_0} \xleftrightarrow[mT_0]{\mathcal{FS}} \delta[k-m]$$



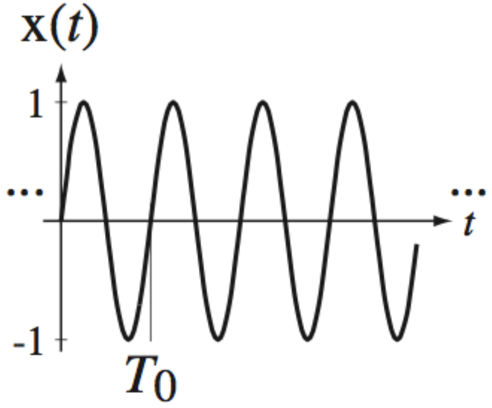
Fourier Transform



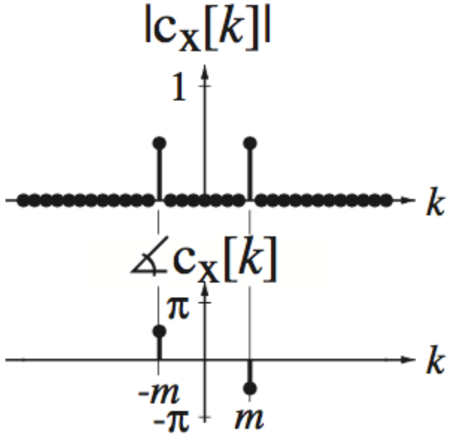
$$\cos(2\pi t/T_0) \xleftrightarrow[mT_0]{\mathcal{FS}} (1/2)(\delta[k-m]+\delta[k+m])$$



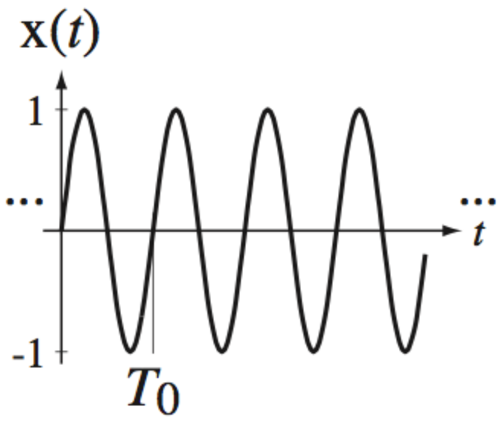
Fourier Transform



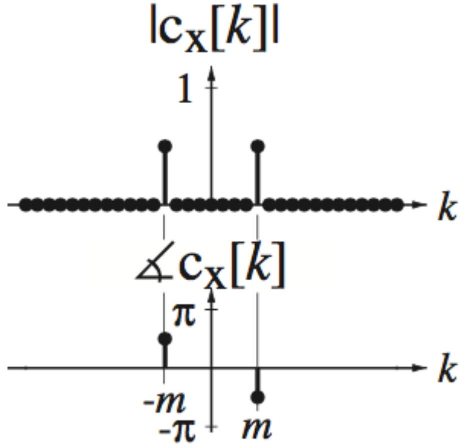
$$\sin(2\pi t/T_0) \xleftrightarrow[mT_0]{\mathcal{FS}} (j/2)(\delta[k+m]-\delta[k-m])$$



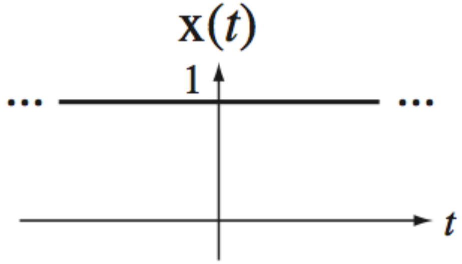
Fourier Transform



$$\sin(2\pi t/T_0) \xleftrightarrow[mT_0]{\mathcal{FS}} (j/2)(\delta[k+m] - \delta[k-m])$$

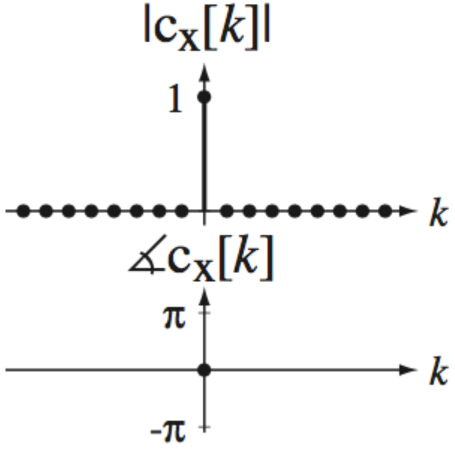


Fourier Transform

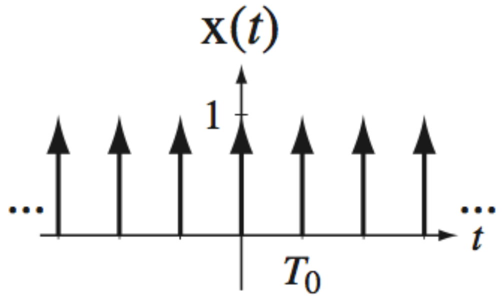


$$1 \xleftrightarrow[T]{\mathcal{FS}} \delta[k]$$

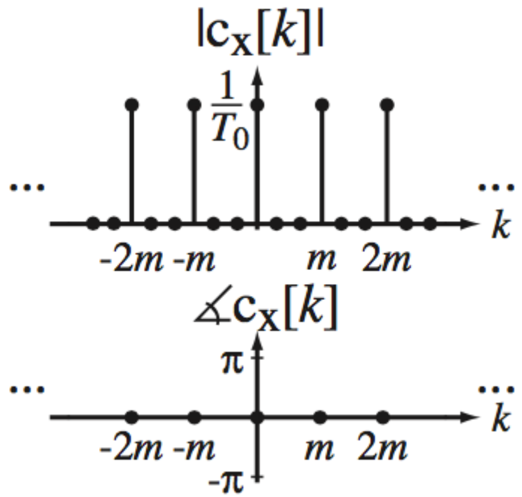
T is arbitrary



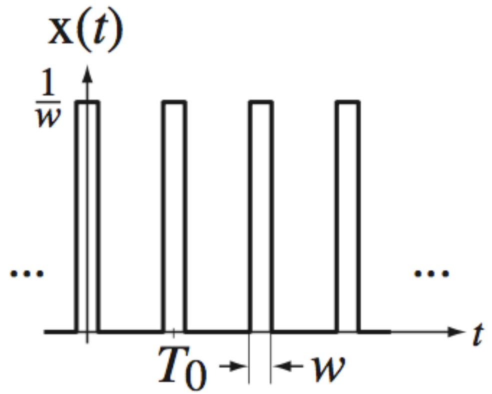
Fourier Transform



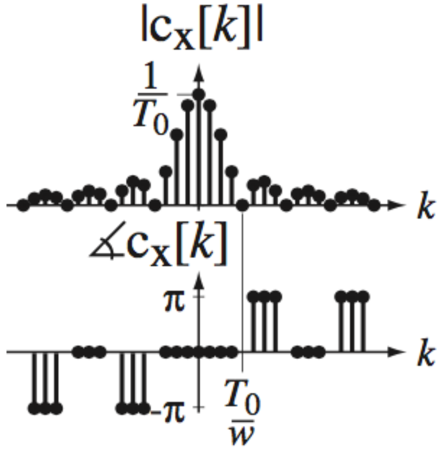
$$\delta_{T_0}(t) \xleftrightarrow[mT_0]{\mathcal{FS}} f_0 \delta_m[k]$$



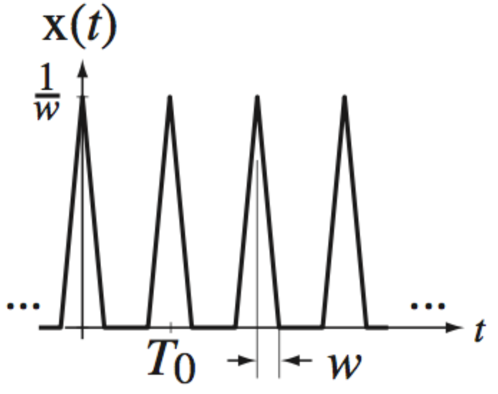
Fourier Transform



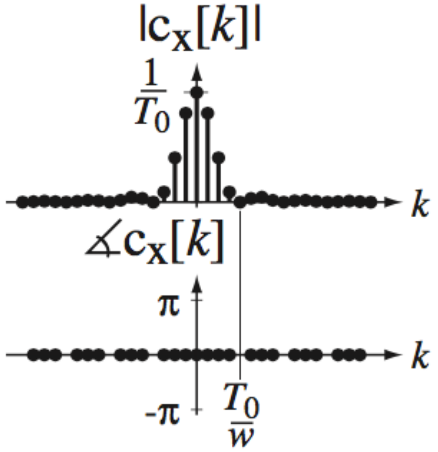
$$(1/w)\,\mathrm{rect}(t/w) * \delta_{T_0}(t) \xleftrightarrow[T_0]{\mathcal{FS}} f_0\,\mathrm{sinc}(wkf_0)$$



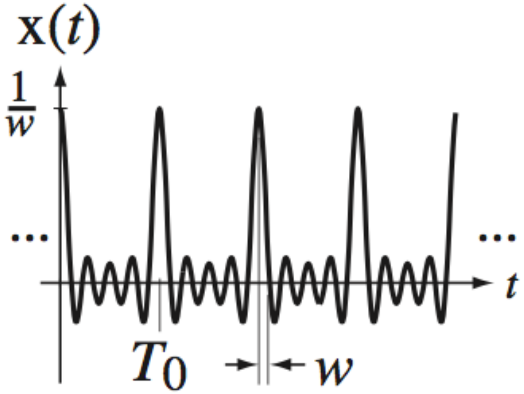
Fourier Transform



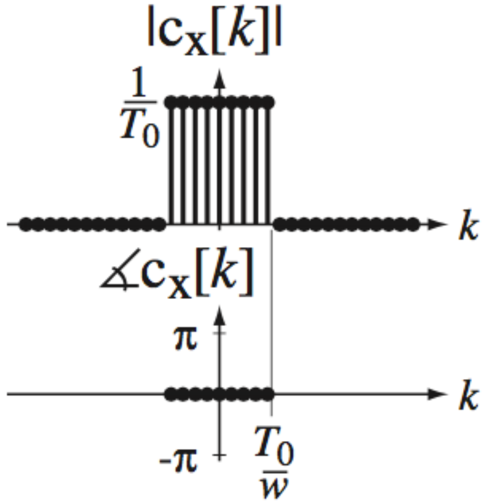
$$(1/w)\,\text{tri}(t/w) * \delta_{T_0}(t) \xleftrightarrow[T_0]{\mathcal{FS}} f_0\,\text{sinc}^2(wkf_0)$$



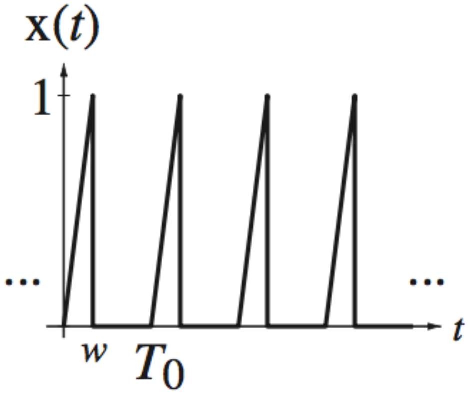
Fourier Transform



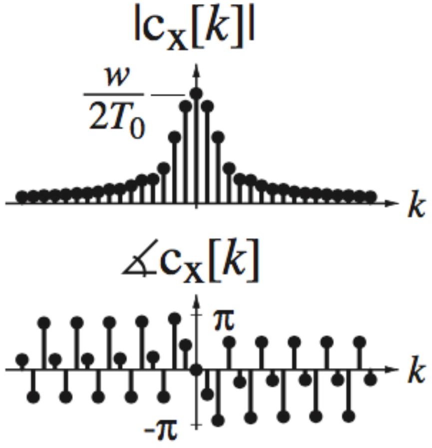
$$(1/w)\,\mathrm{sinc}(t/w) * \delta_{T_0}(t) \xleftrightarrow[T_0]{\mathcal{FS}} f_0\,\mathrm{rect}(wkf_0)$$



Fourier Transform



$$\frac{t}{w}[\mathrm{u}(t)-\mathrm{u}(t-w)] * \delta_{T_0}(t) \xleftrightarrow[T_0]{\mathcal{FS}} \frac{1}{wT_0}\frac{[j(2\pi kw)/T_0+1]e^{-j(2\pi kw/T_0)}-1}{(2\pi k/T_0)^2}$$



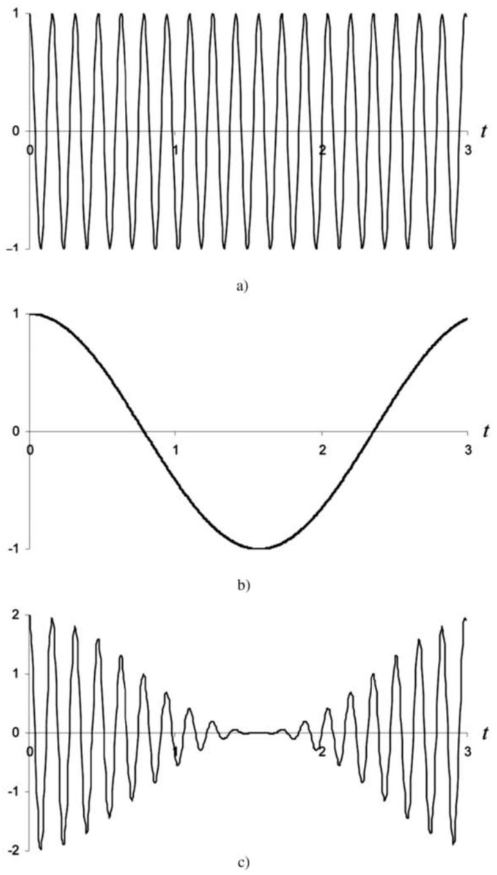
(Modulation)

- The Complex Representation of a Bandpass Signal

$$\exp(jx) = \cos(x) + j \cdot \sin(x)$$

$$\begin{aligned} s(t) &= A(t) \cdot \cos(\omega_c t + \phi(t)) = \mathrm{Re}\{A(t) \cdot \exp(j(\omega_c t + \phi(t)))\} \\ &= \mathrm{Re}\{A(t) \cdot \exp(j\phi(t)) \cdot \exp(j\omega_c t)\} = \mathrm{Re}\{b(t) \cdot \exp(j\omega_c t)\} \end{aligned}$$

(Modulation)

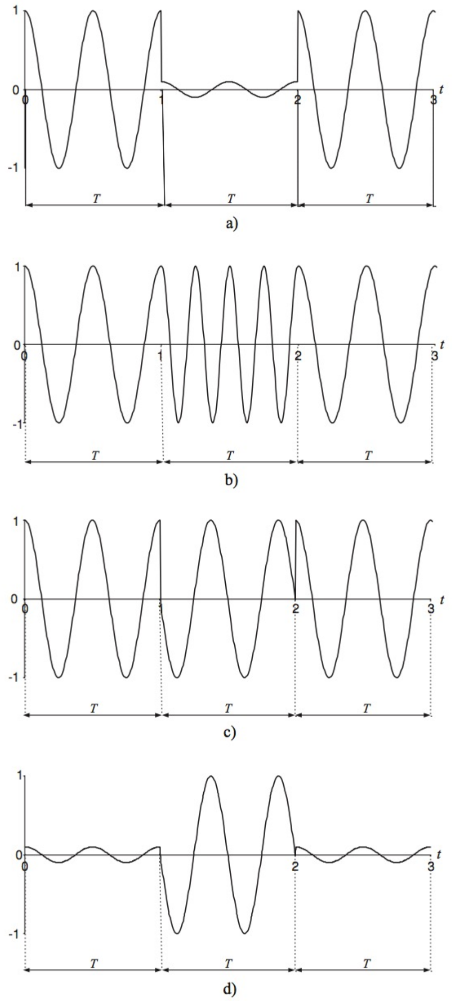


Amplitude modulation

Carrier wave

Modulation bandpass signal

(Modulation)



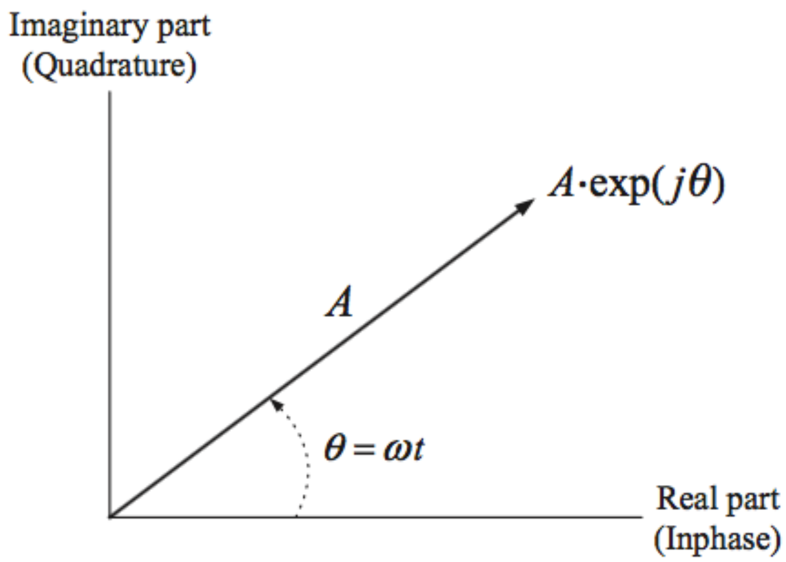
ASK

FSK

PSK

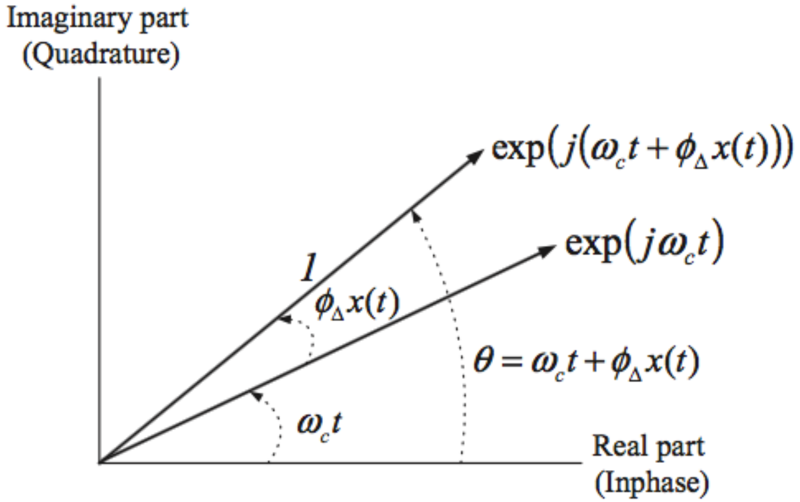
ASK/PSK

Phasor representation of signal



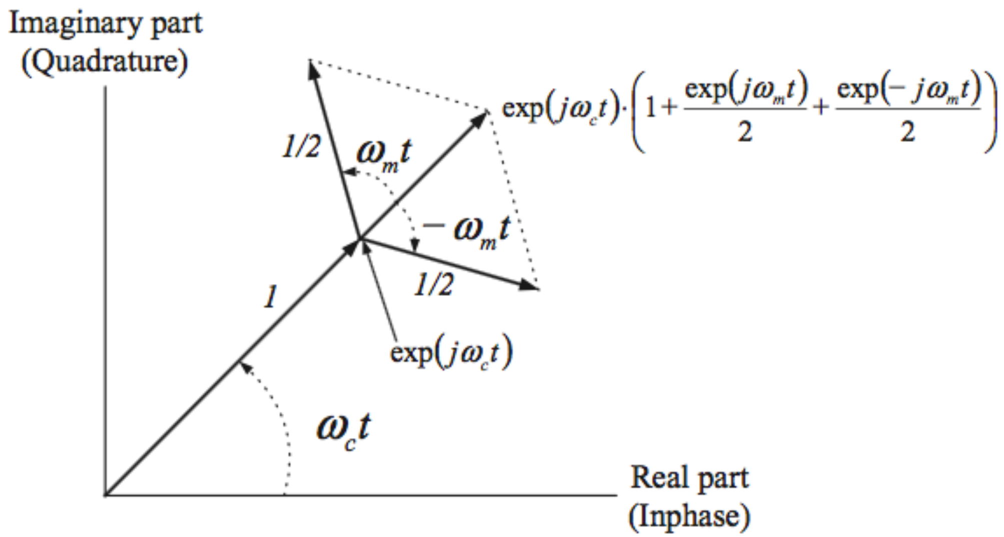
$$s_{PM}(t) = \cos(\omega_c t + \phi_\Delta x(t)) = \mathrm{Re}\{\exp(j\phi_\Delta x(t)) \cdot \exp(j\omega_c t)\}$$

Phasor representation of phase modulation



$$s_{AM}(t) = \cos(\omega_c t) + \cos(\omega_c t) \cdot \cos(\omega_m t) = \cos(\omega_c t) + \frac{\cos((\omega_c + \omega_m)t)}{2} + \frac{\cos((\omega_c - \omega_m)t)}{2}$$

Phasor representation of amplitude modulation



Phasor representation of amplitude modulation

$$s_{AM}(t) = \mathrm{Re}\left\{\exp(j\omega_c t) + \frac{\exp(j(\omega_c + \omega_m)t)}{2} + \frac{\exp(j(\omega_c - \omega_m)t)}{2}\right\}$$

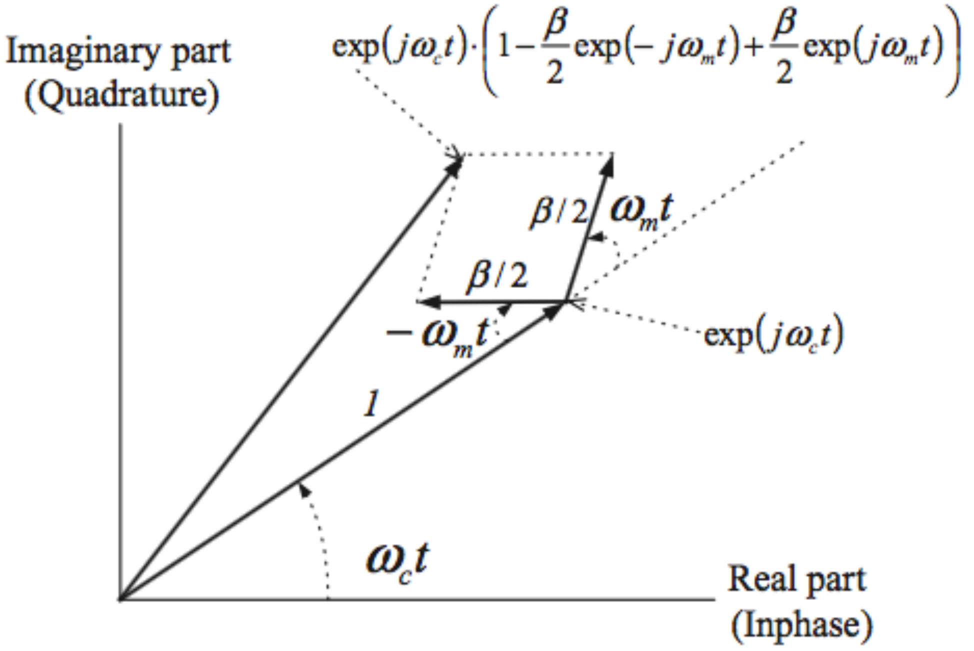
$$s_{AM}(t) = \mathrm{Re}\left\{\exp(j\omega_c t) \cdot \left(1 + \frac{\exp(j\omega_m t)}{2} + \frac{\exp(-j\omega_m t)}{2}\right)\right\}$$

Phasor representation of amplitude modulation

$$s_{FM}(t) = \cos(\omega_c t) - \beta \sin(\omega_c t) \cdot \sin(\omega_m t) = \cos(\omega_c t) - \beta \frac{\cos((\omega_c - \omega_m)t)}{2} + \beta \frac{\cos((\omega_c + \omega_m)t)}{2}$$

$$s_{FM}(t) = \mathrm{Re}\left\{\exp(j\omega_c t) \cdot \left(1 - \frac{\beta}{2}\exp(-j\omega_m t) + \frac{\beta}{2}\exp(j\omega_m t)\right)\right\}$$

Phasor representation of frequency modulation



Phase shift keying (PSK)

$$s_i(t) = h_{Tx}(t)\cos(\omega_c t + \phi_i), \, i = 1, \ldots, M$$

$$\phi_i = \frac{2\pi i}{M}$$

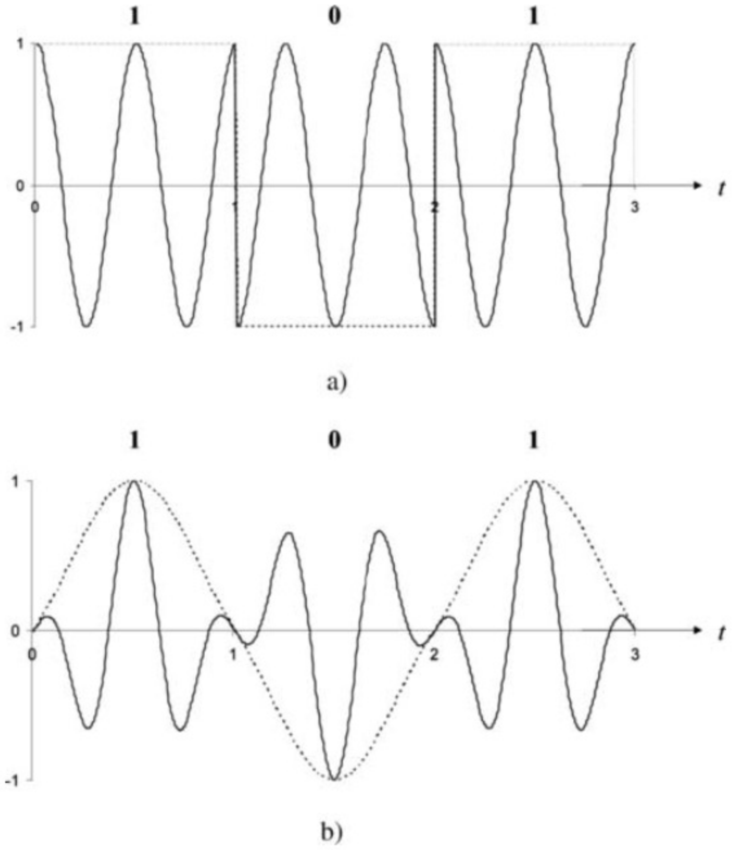
$$s_i(t) = \mathrm{Re}\{\exp(j\phi_i)h_{Tx}(t)\exp(\omega_c t)\}, \, i = 1, \ldots, M$$

Binary phase shift keying (BPSK)

$$s_i(t) = \begin{cases} h_{Tx}(t)\cos(\omega_c t), & \text{if 1 is emitted} \\ -h_{Tx}(t)\cos(\omega_c t), & \text{if 0 is emitted} \end{cases}$$

$$s_i(t) = \begin{cases} h_{Tx}(t)\cos(\omega_c t), & \text{if 1 is emitted} \\ -h_{Tx}(t)\cos(\omega_c t), & \text{if 0 is emitted} \end{cases}$$

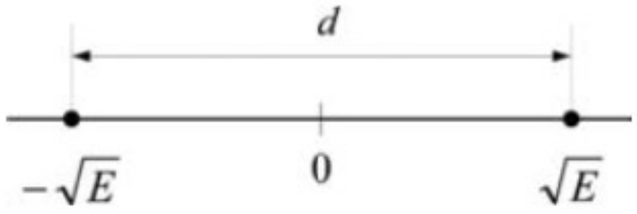
Binary phase shift keying (BPSK)



Rectangular pulse shaping

Raised-cosine pulse shaping

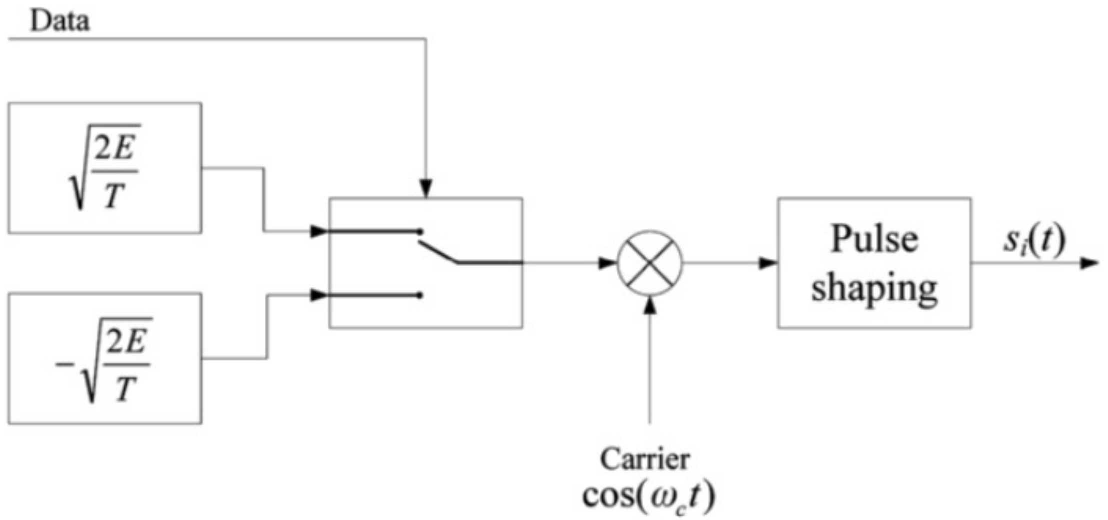
Binary phase shift keying (BPSK)



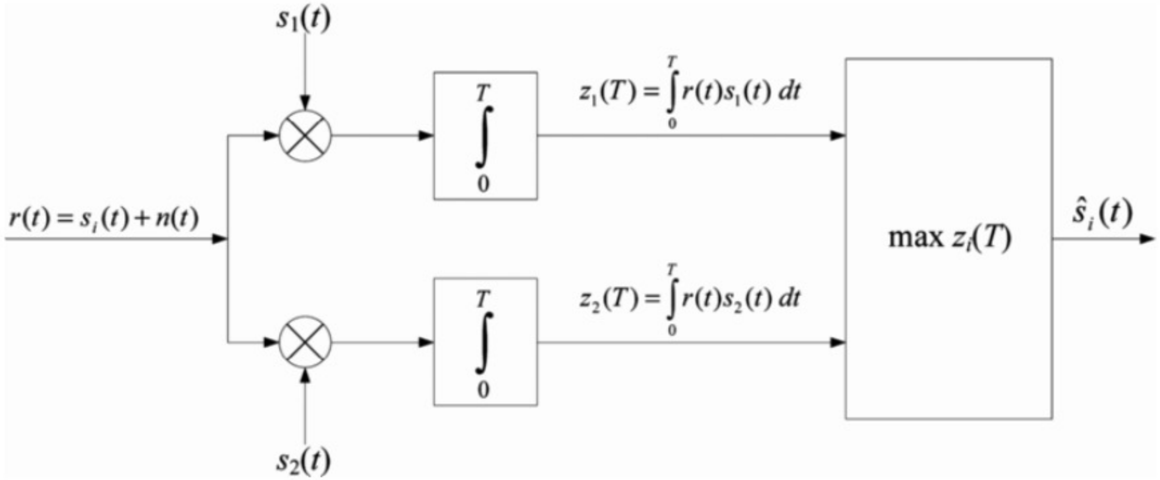
Constellation of BPSK signals

$$z(T) = a_i(T) + n_1(T)$$

BPSK modulator



BPSK demodulator (Two correlators)



BPSK demodulator (Two correlators)

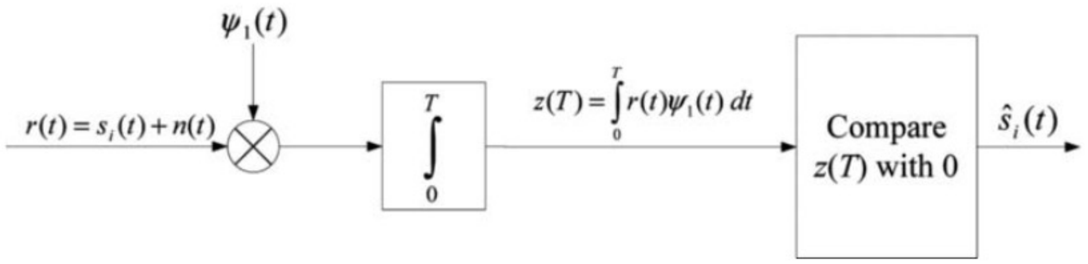
$$p(z|s_1) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{z-\sqrt{E}}{\sigma}\right)^2\right)$$

$$p(z|s_2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{z+\sqrt{E}}{\sigma}\right)^2\right)$$

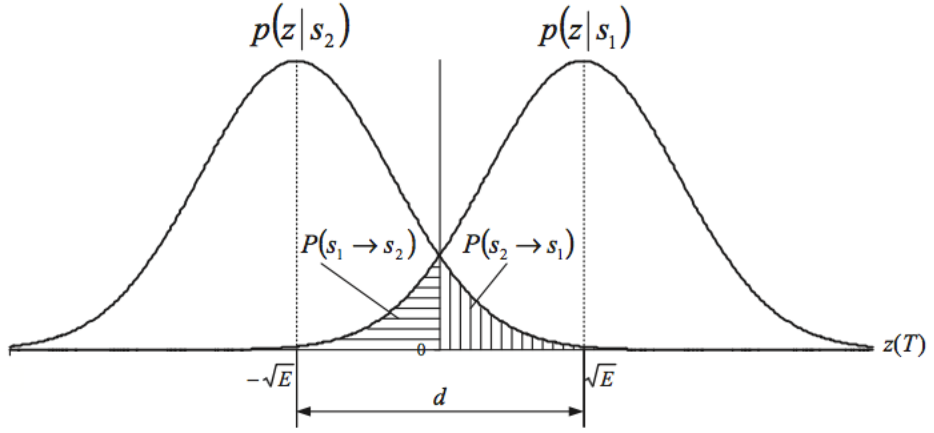
$$P(s_1 \to s_2) = \int_{-\infty}^{0} p(z|s_1)\,dz$$

$$P(s_2 \to s_1) = \int_{0}^{\infty} p(z|s_2)\,dz$$

BPSK demodulator (Two correlators)



1 correlator



Conditional pdfs of the correlator output

หนังสืออ้างอิง (Ref.)
Required Text

- Michael J.ROBERTS, “Signals and Systems Analysis Using Transform Methods and MATLAB” MaGraw-Hill 2nd ,2012
- Evgenii Krouk and Sergei Semenov, “Modulation and coding techniques in wireless communications” John Wiley & Sons 1st , 2011.